# Math 53 Discussion Problems Nov 19

1. Find the line integrals of $\mathbf{F}$ over the given paths.

   (a) $\mathbf{F}(x, y) = \langle x^2, -y \rangle$, along $\mathbf{r}(t) = \langle t^2, t \rangle$, $-1 \leq t \leq 2$

   (b) $\mathbf{F}(x, y) = \langle y, -x \rangle$, counterclockwise along the unit circle from $(1, 0)$ to $(0, 1)$

   (c) $\mathbf{F}(x, y, z) = \langle -4xy, 8y, 2 \rangle$, along $\mathbf{r}(t) = \langle t, t^2, 1 \rangle$, $0 \leq t \leq 2$

   (d) $\mathbf{F}(x, y, z) = \langle xy, yz, xz \rangle$, along the path $C_1 \cup C_2$ consisting of the line segment from $(0, 0, 0)$ to $(1, 1, 0)$ followed by the segment from $(1, 1, 0)$ to $(1, 1, 1)$

   (e) $\mathbf{F}(x, y, z) = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$, along the line segment from $(1, 0, 1)$ to $(1, \frac{\pi}{2}, 0)$ (Hint: use the fundamental theorem for line integrals)

2. Determine whether each of the following vector fields $\mathbf{F}$ are conservative. For those that are, find a potential function $f$ for $\mathbf{F}$.

   (a) $\mathbf{F}(x, y) = \langle -y, x \rangle$

   (b) $\mathbf{F}(x, y) = \langle \frac{2x}{y}, \frac{1-x^2}{y^2} \rangle$

   (c) $\mathbf{F}(x, y) = \langle ye^x, e^x + e^y \rangle$

   (d) $\mathbf{F}(x, y, z) = \langle z + y, z, y + x \rangle$

   (e) $\mathbf{F}(x, y, z) = \langle e^x \cos y, -e^x \sin y, z \rangle$